Multi-Planar Projection by Fixed-Center Pan-Tilt Projectors

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Abstract

We describe a new steerable projector, whose projection center precisely corresponds with its rotation center, which we call a "fixed-center pan-tilt (FC-PT) projector." This mechanism allows it be set up more easily to display graphics precisely on the planes in the environment than for other steerable projectors; wherever we would like to display graphics, all we have to do are locating the FC-PT projector in the environment, and directing it to the corners of the planes whose 2D sizes have been measured. Moreover, as the FC-PT projector can recognize automatically whether each plane is connected to others, it can display visual information that lies across the boundary line of two planes in a similar way to a paper poster folded along the planes.

1. Introduction

Displaying visual information wherever in the real world has recently been attracting increasing attention in many research areas; human interfaces, ubiquitous computing, and computer vision, for examples. And also there are many approaches to display them. The most popular ways are using display devices, such as displays embedded in the environments and head-mounted displays. As they are getting thiner, smaller and lighter, they are expected to be used in more situations in our daily lives. However, they have some disadvantages. In the case of the embedded one, its drawable area is so restricted (only on its screen, of course) that we need a lot of such displays to cover the environment, and of course it costs a lot. In the case of the head-mounted one, it is in fact very convenient because it is so small that it can be worn any time and can be used wherever its user is existing. However, to accomplish the geometrical consistency between the visual information and the environment, we have to measure the 3D structure of the environment and its 3D position and orientation precisely, which are too hard to be measured precisely.

On the other hand, there is also way by a projector. This way has the advantages that they can be easily located afterward in the environment, and that graphics displayed in the
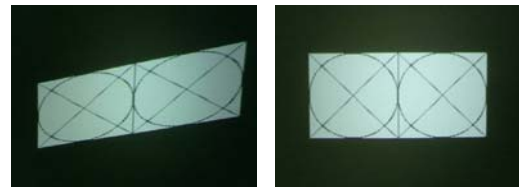
(a) distorted graphic   (b) undistorted graphic
Figure 1: Distortion of projected graphics

real world can be observed easily for us, especially compared with the small head-mounted displays. Moreover, if it has rotatable mechanism it can display the graphics on a wider are in the environment, and in addition to it, if there exist multiple such rotatable projectors, they may display the graphics on almost all surfaces in the environment, and can display larger images by projecting simultaneously.

Just by rotating the projector, projected graphics are distorted according to the relative orientation between the projector and the projected plane, as shown in Figure 1(a). To display the undistorted graphics on the plane (as shown in Figure 1(b)), we have to input the inversely distorted graphics to the projector. Especially in the case of the rotatable projector, as its orientation can change continuously, the inversely distorted image on its input image has to be generated precisely in real time. In this paper, the rotatable projector is designed so that its projection center precisely corresponds with its rotation center. We call such a steerable projector a "fixed-center pan-tilt (FC-PT) projector." With the FC-PT mechanism, set up and control of the FC-PT projector are much easier than for other usual steerable projectors, as described in Section 2. After calibration of the FC-PT projector, which is described in Section 4, we simply need to place it in the environment, and direct it to the corners of planes whose 2D areas have been measured. And after these operations, we can obtain graphical information at any position where we would like to display it on the planes without distortion. We describe this calibration method in Section 4, and the projection method in Section 5.

Moreover, using the directions of the planes' corners, the FC-PT projector recognizes automatically whether each
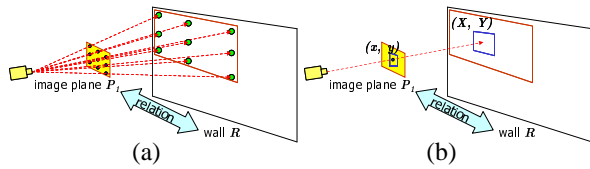
Figure 2: Projection using a fixed projector

Figure 3: Projection in a different direction using a PT projector

Figure 4: Projection using the FC-PT projector

plane connects to others. Therefore, it can display a graphic that lies across the boundary line of two planes in the same way as a paper poster folded along the planes. We describe this function in Section 6.

There are several related works: In [1], they developed a hand-held projector with a camera. Although it can display undistorted graphics wherever the user hope, known patterns have to be projected preliminary and a partial 3D structure of the environment has to be reconstructed every time we place the projector. In [2], they developed a steerable projector. However, as it is not designed in order that the projection center corresponds precisely at its rotation center, we have to project preliminary the patterns on every plane in the environment. In another study [3], graphics can be displayed on multiple planes; however, the projector is fixed.

2. The Advantages of the FC-PT Projector

2.1. The Fixed Projector

Only when the projector is located perpendicular to the plane, we can display graphics of the same shape as the input of the projector. Unless this condition is achieved, the graphics projected on the plane is distorted according to the projector's orientation. And the sizes of the graphics are changed according to the relative position of the projector and the plane. Because of these, to display graphics whose size, shape and position we specify, we have to preliminary calculate the relation between the input image P_1 of the projector and the plane R. To obtain this relation, known patterns (grid patterns, for example) are generated on the image plane P_1, projected onto the wall R, and measured (Figure 2(a)). As the relation between these coordinate systems is described by a 3×3 homography matrix [5], a minimum of four pairs of coordinates are needed to obtain the relation before the projector can display undistorted images on the wall (Figure 2(b)).

2.2. The Pan-Tilt Projector

To display graphics on a wider area on a plane or on more planes in the environment, the projector is placed on a rotatable stage that pans and tilts electrically. We call such a projector a "pan-tilt projector." As with the fixed projector, the relation between the image plane and the wall must be obtained. However, for t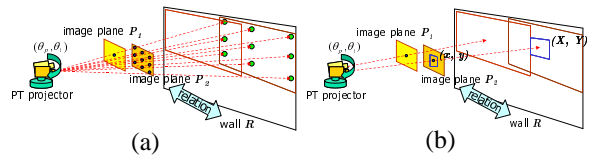he pan-t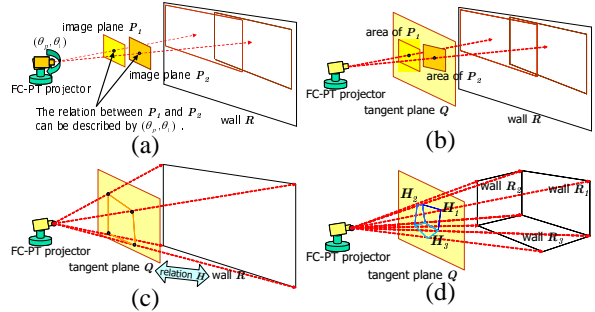ilt projector, this is more complicated because the image plane moves in space according to the change of its orientation.

In Figure 2.2, for example, if we rotate the projector to a different direction, as a result of which the image plane moves from P_1 to P_2, we now need the new relation between P_2 and R (Figure 2.2(a)). Then, similarly, we have to repeat the process again, as for the initial direction, after which the projector can display undistorted graphics on the wall (Figure 2.2(b)). In the case of the pan-tilt projectors, because the relative positions of the projection center and the rotation center are unknown and hard to measure, relations between the image planes of different orientations, for example the relation between P_1 and P_2, can not be calculated correctly. Therefore, to be able to display graphics precisely wherever on R we would like to display it, we can not avoid executing the same process for every orientation as for the initial orientation. When we would like to display graphics on multiple planes in the environment, of course, the process has to be done more times.

In addition to the problem above, there is also another problem that moving graphics, which force it to rotate while projecting, can not be displayed by the methods. This is because the projector can take only specific directions which have been taken when it has displayed the known patterns to the plane.

2.3. The FC-PT Projector

In this study, we present the FC-PT projector, whose projection center precisely corresponds with the rotation center in space. This mechanism is similar to the FV-PTZ camera [4]. The FC-PT projector can overcome the problem discussed in Section 2.2.

With the FC-PT projector, since the projection center does not move according to changes of direction, the rela-

tion between the image planes at different directions can be calculated only from the rotation angles (Figure 4(a)). Here, we introduce a plane to integrate all the image planes of different orientations. We call it the "tangent plane" (Figure 4(b)). The image plane of the FC-PT projector is projected on this tangent plane, and this projected region (called "projectable area" in Section 4), and the position and shape of this projectable area on the tangent plane can be calculated. Therefore, as a result, we can treat the image coordinates for the arbitrary directions on the tangent plane. The calculation for this description calibrates the FC-PT projector, and its details are described in Section 4.

After calibration, the relation between the tangent plane and the plane in the environment has to be obtained. The tangent plane does not move according to the rotation, therefore, we need only to specify the information of directions of four points on the plane (its four corners, for example) (Figure 4(c)). Moreover, this operation can be extended easily to multiple planes in the environment; not only a wall but also other walls, floors, ceilings and so on (Figure 4(d)).

3. Implementation of the FC-PT Projector System

3.1. Fixing the Projection Center

We prepare a rotatable stage that pans and tilts electrically, as shown in Figure 5. The pan and tilt axes must cross each other, and the position of their intersection is fixed independently of the stage's orientation. To construct the FC-PT projector, its projection center must precisely correspond with this intersection. However, the complexity of obtaining the position of the projection center in space is such that we cannot directly fix the projection center precisely at the rotation center. Here, we apply a two-step method for constructing the FC-PT projector: rough estimation of the projection center's position, and precise fixing.

First, we roughly estimate the position of the projection center. A planar board is placed vertically and then horizontally in front of the projector as shown in Figure 6, and the boundary lines of the projected light are recorded. We then calculate the intersection of these lines, and treat the intersection as the roughly estimated position of the projection center.

Next, the projector is placed on the rotatable stage so that the roughly estimated position of the projection center corresponds with the rotation center, as shown in Figure 7. The projector can be three-dimensionally repositioned on the stage along the x, y and z axes. There are two screens in front of the projector, and the nearer one (screen 1) has a small hole. When the projector is turned on, only the light that goes through this hole reaches at the further screen (screen 2). If the projection center is precisely at the rotation center, the projected point on screen 2 does not move
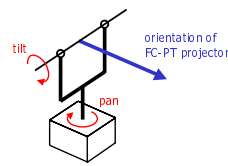

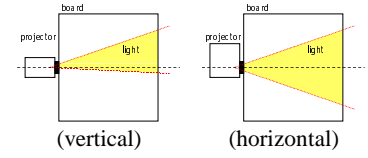Figure 5: Structure of rotatable stage

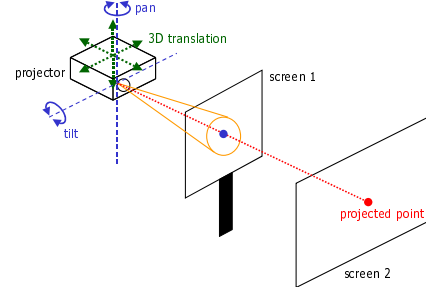Figure 6: Rough estimation of projection center

Figure 7: Precise fixing

even when the stage pans and tilts in any direction. This is because the light always originates from the same point. Inversely, the further the projection center is from the rotation center, the more the projected point moves. Therefore, the projector is continuously repositioned on the stage to search for the best position, where the projected point does not move at all.

3.2. System Configuration

By the process described in Section 3.1, the projection center is fixed precisely at the rotation center of the stage. A PC is then connected to the FC-PT projector; input images for the projector are transmitted from the PC, which also captures and controls orientation of the rotatable stage. We refer to this configuration as the "FC-PT projector system."

4. Calibration of the FC-PT Projector

For calibrating the FC-PT projector, we define the "tangent plane" as the plane that exists in front of the projector and is parallel to the pan and tilt axes of the stage at the initial orientation, as shown in Figure 8. We further assume that the distance between the tangent plane and the projection center is equal to 1. As is clear from the definition, the tangent plane is fixed independently of the orientation of the FC-PT projector. In this section, we show that we can easily manipulate the image coordinates on this tangent plane at any orientation of the FC-PT projector.

First, we set the FC-PT projector to be at the initial orientation $\theta_p = \theta_t = 0$, where θ_p and θ_t denote the pan and tilt angles of the projector respectively. Grid points (x_i, y_i) $(i = 1, 2, 3, \cdots)$ each of whose coordinates on the image plane P are known, are projected at (X_i, Y_i) on the plane R in the environment. Here, these coordinates fulfill the
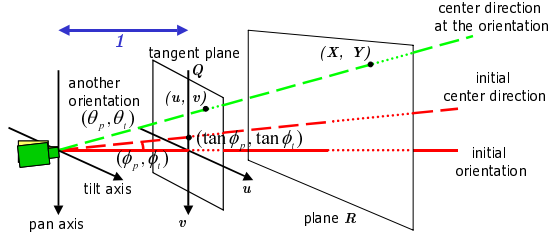
Figure 8: Tangent plane

equation below:

$$\begin{pmatrix} X \\ Y \\ 1 \end{pmatrix} = H_{PR} \begin{pmatrix} x \\ y \\ 1 \end{pmatrix}, \qquad (1)$$

where H_{PR} denotes a 3×3 matrix having 8 degrees of freedom, which is called the homography matrix [5]. We can calculate H_{PR} linearly from a minimum of four pairs of (x_i, y_i) and (X_i, Y_i)[1]

Next, we project a point at the center of the image plane, onto the plane in the environment. We call this ray to the center of the image plane the "center direction." In the case of an ordinary projector, the center direction is different from the orientation of the projector, so that the intersection point (u_c, v_c) of the center direction and the tangent plane is not $(0, 0)$ but $(\tan \phi_p, \tan \phi_t)$ on the tangent plane as shown in Figure 8, where ϕ_p, ϕ_t denote the difference between the front direction and the center direction of the projector.[2] Therefore, when we rotate the projector to (θ_p, θ_t), the intersection (u, v) on the tangent plane is calculated as follows:

$$\begin{pmatrix} u \\ v \\ 1 \end{pmatrix} \sim \begin{pmatrix} \lambda u \\ \lambda v \\ \lambda \end{pmatrix} = Rot_{(y, \theta_p)} Rot_{(x, \theta_t)} \begin{pmatrix} \tan \phi_p \\ \tan \phi_t \\ 1 \end{pmatrix}, \qquad (2)$$

where $Rot_{(w, \theta)}$ denotes a matrix which rotates a coordinate at an angle θ around the w axis, and "$\sim$" denotes a equivalence in the projective geometry. We next rotate the projector to other orientations $(\theta_{pi'}, \theta_{ti'})$ $(i' = 1, 2, 3, \cdots)$, and project the center point of the image plane. For each orientation, we measure the projected point $(X_{i'}, Y_{i'})$ on the plane in the environment[3], and calculate the intersection $(u_{i'}, v_{i'})$ on the tangent plane from Equation (2). Note that the projection center is always fixed in the case of the FC-PT projector, so that $(u_{i'}, v_{i'})$ on the tangent plane is

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[1]To obtain (X_i, Y_i), we use a camera. The image captured by the camera is undistorted using Tsai's method [6]. On the plane we display four points whose metric positions are known. By capturing the projected points and these four points, we can obtain (X_i, Y_i).

[2]ϕ_p and ϕ_t are obtained from a speculation document of the projector. However, these parameters of the speculation usually include errors. It is desirable that a method to calculate them precisely is developed, and the calculated parameters are used as ϕ_p and ϕ_t. The development of this method is included in our future works.

[3]In the same way as measuring (X_i, Y_i), we can measure them by the camera.

perspectively projected to $(X_{i'}, Y_{i'})$ on the plane in the environment. Therefore, $(u_{i'}, v_{i'})$ and $(X_{i'}, Y_{i'})$ also fulfill the following equation:

$$\begin{pmatrix} X \\ Y \\ 1 \end{pmatrix} = H_{QR} \begin{pmatrix} u \\ v \\ 1 \end{pmatrix}. \qquad (3)$$

In a similar way to Equation (1), from a minimum of four pairs of $(u_{i'}, v_{i'})$ and $(X_{i'}, Y_{i'})$, we can calculate H_{QR} linearly. From Equations (1) and (3), we can obtain the equation below:

$$\begin{pmatrix} u \\ v \\ 1 \end{pmatrix} = H_{QR}^{-1} H_{PR} \begin{pmatrix} x \\ y \\ 1 \end{pmatrix} = H_{PQ} \begin{pmatrix} x \\ y \\ 1 \end{pmatrix}. \qquad (4)$$

Because P denotes the image plane at the initial orientation, by substituting the coordinates of the four apices and the center of the image plane $((\pm 1024/2, \pm 768/2)$ and $(0, 0)$ in XGA, for example) to (x, y) in Equation (4), we can obtain the directions of these apices $(u_j^{(init)}, v_j^{(init)})$ $(j = 1, \cdots, 4)$ and the center $(u_c^{(init)}, v_c^{(init)})$, where $(u_j^{(init)}, v_j^{(init)})$ determine the projectable area on the tangent plane at the initial orientation, and $(u_c^{(init)}, v_c^{(init)})$ denotes the center of the projectable area.

By using the projectable area and the center on the tangent plane at the initial orientation, we get able to calculate those at arbitrary orientations easily. This is because the relative directions between 3D vectors $(u_j, v_j, 1)^T$ and $(u_c, v_c, 1)^T$ in arbitrary orientation remain unchanged from $(u_j^{(init)}, v_j^{(init)}, 1)^T$ and $(u_c^{(init)}, v_c^{(init)}, 1)^T$. Therefore, we can calculate the projectable area at any orientation simply by rotation of the vectors.

For example, when would like to obtain the projectable area $(u_j^{(new)}, v_j^{(new)})$ from the center direction $(u_c^{(new)}, v_c^{(new)})$ they can be calculated by the following process.

When we direct the center direction of the FC-PT projector to $(u_c^{(new)}, v_c^{(new)})$, its orientation $(u_o^{(new)}, v_o^{(new)})$ is calculated as follows:

$$\begin{pmatrix} u_o^{(new)} \\ v_o^{(new)} \\ 1 \end{pmatrix} \sim Rot_{(y, -\phi_p)} Rot_{(x, -\phi_t)} \begin{pmatrix} u_c^{(new)} \\ v_c^{(new)} \\ 1 \end{pmatrix}. \qquad (5)$$

When we rotate the FC-PT projector to $(\theta_p^{(new)}, \theta_t^{(new)})$, its orientation is calculated as follows:

$$\begin{pmatrix} u_o^{(new)} \\ v_o^{(new)} \\ 1 \end{pmatrix} \sim Rot_{(y, \theta_p^{(new)})} Rot_{(x, \theta_t^{(new)})} \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

$$= \begin{pmatrix} \sin \theta_p^{(new)} \cos \theta_t^{(new)} \\ -\sin \theta_t^{(new)} \\ \cos \theta_p^{(new)} \cos \theta_t^{(new)} \end{pmatrix}. \qquad (6)$$
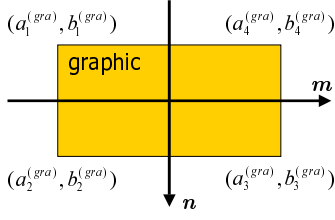
Figure 9: Coordinates of graphic

As $\left(u_o^{(new)}, v_o^{(new)}\right)$ has been calculated in Equation (5), we can calculate $\left(\theta_p^{(new)}, \theta_t^{(new)}\right)$ from Equation (6).

$$
\begin{aligned}
\theta_p^{(new)} &= \tan^{-1} u_o^{(new)}, &(7)\\
\theta_t^{(new)} &= -\tan^{-1}(v_o^{(new)} \cos \theta_p^{(new)}). &(8)
\end{aligned}
$$

Once we have obtained $\left(\theta_p^{(new)}, \theta_t^{(new)}\right)$, we can easily obtain the apices of the projectable area on the tangent plane at this orientation:

$$
\begin{pmatrix} u_j^{(new)} \\ v_j^{(new)} \\ 1 \end{pmatrix} \sim Rot_{(y,\theta_p^{(new)})} Rot_{(x,\theta_t^{(new)})} \begin{pmatrix} u_j^{(init)} \\ v_j^{(init)} \\ 1 \end{pmatrix}. \quad (9)
$$

5. Projection onto a Plane

After calibration, the relation between the tangent plane Q and a plane R_1 on which we will display graphics is needed. As this relation is described as a homography matrix H_{QR1} and can be calculated only by Equations (2) and (3) with four pairs of coordinates on these planes, we need to measure the orientations of the center direction to the four corners of the plane R_1, whose 2D metric coordinates have been obtained. The FC-PT projector's orientations can be obtained by manually directing its center direction to these corners. Hereafter, obtaining the relation between Q and R with the corners' coordinates is referred to as "plane registration."

We now describe the calculation for the FC-PT projector to display graphical information without distortion on the plane R_1 in the environment. Assume, for example, that a rectangular graphic is displayed whose apices are $\left(a_k^{(gra)}, b_k^{(gra)}\right) (k = 1, \cdots, 4)$, defined by their own metric coordinate system as shown in Figure 9. To display the rectangle precisely, we have to calculate the apices on the image plane $\left(x_k^{(ans)}, y_k^{(ans)}\right)$ according to the orientation of the FC-PT projector $\left(\theta'_p, \theta'_t\right)$, as shown in Figure 10. As we have obtained the relation H_{QR1}, this can be solved as follows.

First, the system records the orientation $\left(\theta'_p, \theta'_t\right)$ of the FC-PT projector, and describes the center direction on the tangent plane as (u_c, v_c) by Equation (2). Next, (u_c, v_c) is projected onto a plane in the environment. The projected
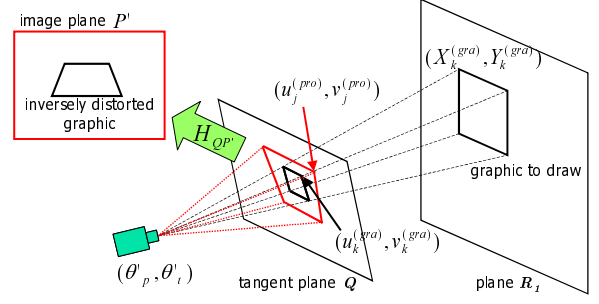

Figure 10: Process for a plane

point (X_c, Y_c) is calculated by:

$$
\begin{pmatrix} X_c \\ Y_c \\ 1 \end{pmatrix} = H_{QR1} \begin{pmatrix} u_c \\ v_c \\ 1 \end{pmatrix}. \quad (10)
$$

The graphic is displayed at (X_c, Y_c) on the plane in the environment, and the four apices of the graphic are calculated as follows:

$$
\begin{aligned}
X_k^{(gra)} &= a_k^{(gra)} + X_c, \\
Y_k^{(gra)} &= b_k^{(gra)} + Y_c. &(11)
\end{aligned}
$$

The for apices are then projected onto the tangent plane:

$$
\begin{pmatrix} u_k^{(gra)} \\ v_k^{(gra)} \\ 1 \end{pmatrix} = H_{QR1}^{-1} \begin{pmatrix} X_k^{(gra)} \\ Y_k^{(gra)} \\ 1 \end{pmatrix}. \quad (12)
$$

As described in Section 4, when we have obtained $\left(\theta'_p, \theta'_t\right)$ we can also easily obtain the projectable area on the tangent plane at this orientation from Equation(9), as follows:

$$
\begin{pmatrix} u_j^{(pro)} \\ v_j^{(pro)} \\ 1 \end{pmatrix} \sim Rot_{(y,\theta'_p)} Rot_{(x,\theta'_t)} \begin{pmatrix} u_j^{(init)} \\ v_j^{(init)} \\ 1 \end{pmatrix}. \quad (13)
$$

From Equations (12) and (13), we have described the projectable area and the graphic on the tangent plane in the orientation $\left(\theta'_p, \theta'_t\right)$. We next have to obtain the apices of the graphic on the image plane P' in this orientation to project to the plane R_1. As $\left(u_j^{(pro)}, v_j^{(pro)}\right)$ correspond to the apices $((\pm 1024/2, \pm 768/2)$ in XGA) on P', we can calculate a homography matrix $H_{QP'}$ that transforms the coordinates on Q to those on P'. we can obtain $\left(x_k^{(ans)}, y_k^{(ans)}\right)$, which defines the graphic that should be input to the projector, by applying $H_{QP'}$ to $\left(u_k^{(gra)}, v_k^{(gra)}\right)$ as shown in follows:

$$
\begin{pmatrix} x_k^{(ans)} \\ y_k^{(ans)} \\ 1 \end{pmatrix} = H_{QP'} \begin{pmatrix} u_k^{(gra)} \\ v_k^{(gra)} \\ 1 \end{pmatrix}. \quad (14)
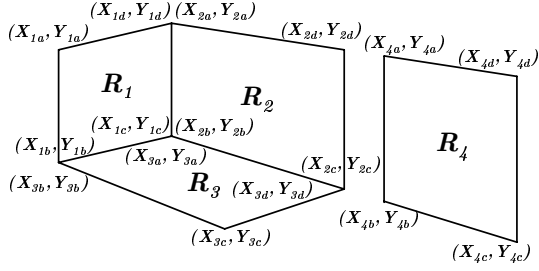$$

Figure 11: Multiple planes in environment

Plane	(u, v)	(x, y)
R_1	(u_{1a}, v_{1a})	(X_{1a}, Y_{1a})
	(u_{1b}, v_{1b})	(X_{1b}, Y_{1b})
	(u_{1c}, v_{1c})	(X_{1c}, Y_{1c})
	(u_{1d}, v_{1d})	(X_{1d}, Y_{1d})
R_2	(u_{2a}, v_{2a})	(X_{2a}, Y_{2a})
	(u_{2b}, v_{2b})	(X_{2b}, Y_{2b})
	(u_{2c}, v_{2c})	(X_{2c}, Y_{2c})
	(u_{2d}, v_{2d})	(X_{2d}, Y_{2d})
R_3	(u_{3a}, v_{3a})	(X_{3a}, Y_{3a})
	(u_{3b}, v_{3b})	(X_{3b}, Y_{3b})
	...	...
...	...	...

Table 1: Registration table

6. Projection onto Multiple Planes

There are many planes in the environment, on which it is desirable to be able to display graphics. The FC-PT projector is appropriate for such demands because the projection method for a plane described in Section 5 can easily be extended to that for multiple planes. In fact, as the plane registration can be done by measuring the direction of the four corners of the plane, we can register even multiple planes in the environment. However, in most cases, this is not insufficient. Because the 2D coordinate system of each plane is defined independently, the projector system cannot recognize connections of the planes, and as a result, it cannot precisely display graphics that lie across boundary lines of the planes. In this section, to overcome the problem, we describe a method for displaying graphics which give the same effect as for paper posters folded and pasted across the boundary lines.

6.1. Preparation for Multi-Planar Projection

In an example shown in Figure 11, there are four planes in the environment for example, and some of them are interconnected. Each plane is described by its own 2D metric coordinate system, which can be determined arbitrarily on the plane. In Figure 11, the coordinate system (X_1, Y_1) is used on the plane R_1 and is independent of other coordinate systems (X_l, Y_l) $(l = 2, 3, 4)$. What we have to do is measuring the direction from the FC-PT projector to each corner of the planes, by rotating the projector manually. The directions are obtained as the orientation (θ_p, θ_t), and they are
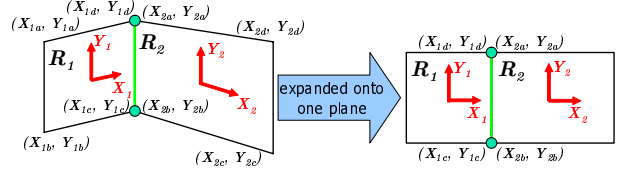

Figure 12: Two planes expanded into one plane

transformed to (u, v) on the tangent plane by Equations (2) and (3). Table 1 shows the list of those coordinates, which is used to register the planes. We call this the "registration table."

6.2. Detection of Relations among Planes

After registration in Section 6.1, the system recognizes the connections of the planes and calculates the relation between each pair of interconnected.

In Figure 11, (X_{1d}, Y_{1d}) and (X_{2a}, Y_{2a}) represent the same point in the environment so that their corresponding points (u_{1d}, v_{1d}) and (u_{2a}, v_{2a}) are the same. (u_{1c}, v_{1c}) and (u_{2b}, v_{2b}) are also the same. From these two points in the environment, the system can recognize that R_1 and R_2 are connected to each other. Using this logic, the system can detect an existing connections by focusing on each pair of planes and searching for (u, v) appearing more than once in the registered data of these planes. If two such points exist, the focused planes are connected to each other. By this process, the system makes a matrix as follows:

$$M = (m_{ij}) = \begin{pmatrix} 0 & 1 & 2 & - \\ 3 & 0 & 4 & - \\ 5 & 6 & 0 & - \\ - & - & - & 0 \end{pmatrix}, \quad (15)$$

where m_{ij} represents whether or not R_i and R_j are connected to each other. For each m_{ij}, "0" means that R_i and R_j are the same plane, and "$-$" means that there is no connection between R_i and R_j. Values larger than "0", mean that the planes are connection. These values are assigned in order $(1, 2, 3, \cdots)$, each of which denotes a unique ID of the pair. We call this matrix the "connection matrix."

Next, the system calculates the relations between the coordinates on the two planes. For each pair of two interconnected planes, by expanding these two planes into one plane as shown in Figure 12, one coordinate is described as the 2D transformation of the other. Therefore, for example, focusing on R_1 and R_2, a point (X_{2n}, Y_{2n}) on R_1 can be expressed by (X_{1n}, Y_{1n}) as follows:

$$\begin{pmatrix} X_{2n} \\ Y_{2n} \end{pmatrix} = \begin{pmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{pmatrix} \begin{pmatrix} X_{1n} \\ Y_{1n} \end{pmatrix} + \begin{pmatrix} t_X \\ t_Y \end{pmatrix}. \quad (16)$$

Since every pair of connected planes has two common points such as (X_{1d}, Y_{1d})-(X_{2a}, Y_{2a}) and (X_{1c}, Y_{1c})-(X_{2b}, Y_{2b}) in the case of the pair R_1 and R_2, the transformation parameters (α, t_X, t_Y) can be calculated linearly by the equation below:

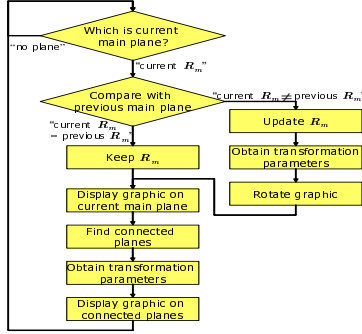| ID	parameters		
1	α_1	t_{X1}	t_{Y1}
2	α_2	t_{X2}	t_{Y2}
3	α_3	t_{X3}	t_{Y3}
...		...	

Table 2: Transformation parameters list

Figure 13: Flowchart of projection

$$\begin{pmatrix} X_{2a} \\ Y_{2a} \\ X_{2b} \\ Y_{2b} \end{pmatrix} = \begin{pmatrix} X_{1d} & -Y_{1d} & 1 & 0 \\ X_{1c} & -Y_{1c} & 1 & 0 \\ Y_{1d} & X_{1d} & 0 & 1 \\ Y_{1c} & X_{1c} & 0 & 1 \end{pmatrix} \begin{pmatrix} \cos\alpha \\ \sin\alpha \\ t_X \\ t_Y \end{pmatrix}. \tag{17}$$

The system calculates the transformation parameters of all pairs of connected planes and saves them with their IDs as shown in Table 2. We call this list the "transformation parameters list."

6.3. Projection Method

The flowchart for multi-planar projection is shown in Figure 13.

First, the system receives the orientation (θ'_p, θ'_t) of the FC-PT projector and describes the center direction on the tangent plane as (u_c, v_c). The system then consults the registration table (Table 1) to find which plane's region (u_c, v_c) is belonging to, and treats this plane as the current main plane R_m.

Next, by the same process as Section 5, the apices of the graphic $G_m : (X_k^{(gra),m}, Y_k^{(gra),m})$ on R_m are calculated by Equation (11). However, in the case of multi-planar projection, G_m must not be directly projected onto Q. This is because we have to consider that every plane in the environment has its finite region and the graphics cannot be displayed out of the region. For example, as shown in Figure 14(a), when the projector is directed to the boundary line of R_m the graphic cannot be completely included in R_m. Therefore, the intersection area T_m of G_m and R_m should be calculated, and be projected to $S_m : (u_k^{(gra),m}, v_k^{(gra),m})$ on Q.

As shown in Figure 14(b), the system then searches for planes R_c connected to the R_m by consulting the connection matrix (Equation (15)). From the connection matrix, the system can find not only planes connected to the main
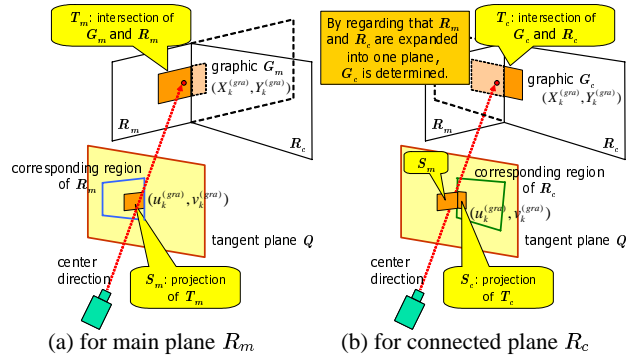

(a) for main plane R_m (b) for connected plane R_c

Figure 14: Projection onto multiple planes

plane, but also the ID of the pair R_m-R_c. It then accesses the transformation parameters list (Table 2) to obtain the appropriate parameters that transform the coordinates on R_m to those on R_c. Having done this, the system can describe the graphic $G_c(X_k^{(gra),c}, Y_k^{(gra),c})$ by the coordinates on R_c. In a Similar way to R_m, the intersection area T_c of G_c and R_c is calculated, and is projected to $S_c : (u_k^{(gra),c}, v_k^{(gra),c})$ on Q. This process is executed for all the planes connected to R_m, and after the process, S_m and all S_c are projected onto the image plane P' of the orientation (θ'_p, θ'_t) by Equation (14).

In addition to the process above, note that the countermeasure against the change of R_m is needed. As the graphic is defined by the coordinates on R_m, when the center direction goes across a boundary line, the orientation of the graphic is changed according to the difference between the coordinates of R_m and R_c, and the graphic cannot move continuously and smoothly. In this system, to overcome this problem, the previous R_m is always preserved, and by comparing the current R_m with the previous R_m the system detects the change of R_m and rotate the graphic appropriately. This additional process is also included in the flowchart shown in Figure 13, and by this process, the graphic can be displayed continuously and smoothly.

6.4. Experimental Results

Figure 15 shows the FC-PT projector. It consists of a projector (V3-131, Plus Vision) and a rotation stage that pans and tilts electrically. We prepared two walls as shown in Figure 16, and measured the lengths of the vertical and horizontal edges of those walls. We then directed the projector to the six points shown in Figure 16 and recorded their directions.

Figure 17 and Figure 18, show the experimental results. We control the projector to always display a $40\text{cm} \times 20\text{cm}$ rectangle on the walls. As shown in (a), (b) and (c) of Figure 17, we can display the rectangle without distortion on each wall; and in (i)–(iv) of Figure 18, we can see the same size rectangle lying across the boundary line of two planes, as
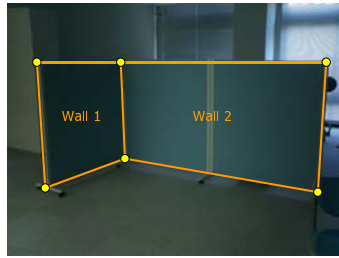
Figure 15: FC-PT projector

Figure 16: Walls for projection

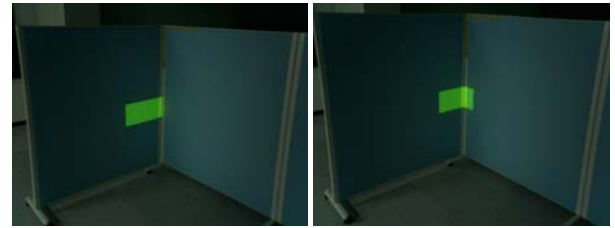(a)   (b)

(c)

Figure 17: Projection on each wall

(i)   (ii)

(iii)   (iv)

Figure 18: Projection on the boundary line

if a paper poster were pasted there folded along the planes. Moreover, as (i)–(iv) represent a time sequence , we confirmed that the rectangle remains continuously undistorted even while moving.
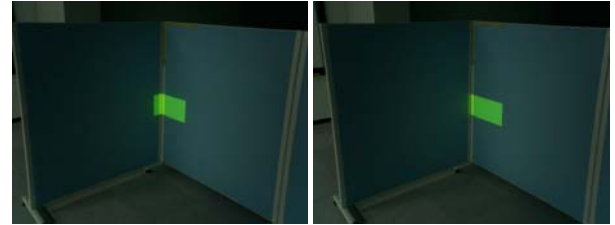
7. Conclusions

We have described a new steerable projector, the FC-PT projector, whose projection center precisely corresponds with its rotation center. We also presented calibration methods for the FC-PT projector. Due to this unique architecture and calibration, we can project visual information at any position on a plane in the environment. Furthermore, we expanded the projection methods to encompass multiple planes, enabling us to use the FC-PT projector for numerous applications.

Future work will include the following:

- In the current implementation, the system needs the metric 2D shape of each plane; these must be measured them before projection. To make the set up more easy, we are now developing methods in collaboration with camera systems to automatically detect and register multiple planes in the real world.

- Although the FC-PT projector can display visual information over a wider area than other fixed projectors, it is inadequate for certain applications. For example, it cannot display visual information on the floor following a person walking a wider area or display images whose size are larger than the capacity of one FC-PT projector. We therefore plan to place multiple FC-PT projectors in the environment, and control them simultaneously. However, when two or more images are projected, they usually do not overlap precisely because of the errors. Overcoming this problem is another important topic.

Acknowledgments

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